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Bachelor Thesis in Mathematics-Economics

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Privatization under a Stackelberg duopoly

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Abstract

This paper presents three models regarding mixed oligopolies in a Stackelberg duopoly. The models are Game Theoretical in approach, drawing lightly on the Contract Theory literature and theories of the firm. In each of the models, the leader company is initially state-owned, but privatization of the firm is under consideration. The three models seek to examine the effect of competition, managerial incentives, and the choice of subsidy scheme upon the tradeoffs arising from the considered privatization.

In the first model, the effect of non-identical goods is examined, and it is shown that the degree of competition faced by the incumbent state-owned company is a relevant factor to consider in regard to privatization.

In the second model, a Manager agent is introduced. It is shown that not all subsidy schemes by the government are effective in promoting productive efficiency in the privatized firm.

In the third model, a model by [Schmidt, 1996] is modified to add the effects of competition upon the decision to privatize.

Contents

Chapter 1

Introduction and survey of the literature

In certain industries, such as railways, there have at times been calls for them to be nationalised (in post-war Europe for example). Within the last few decades though, there has been a push for alternative structures, such as privately held companies providing the same service, but receiving a government subsidy to help them take into account the social benefit of their production. [Kessides, 2005] has a good empirical summary of these tendencies and the empirical facts that have been observed. However, Classical Economics often comes up short in trying to explain key results concerning differences in allocative and productive efficiency.

1.1 Privatization as seen by Classical Economics

Privatization of a company, and the perceived production efficiencies it often achieves, is difficult to explain by Classical Economic theory, which fails to take account of the structure of the firm. Instead, Classical Economists are more concerned with subsidy regimes, how these can be constructed and whether such a thing as an optimal subsidy exists. [Myles, 2002] considers an irrelevance result for a mixed oligopoly, where competing companies have different ownership, and shows that an optimal subsidy level can be chosen where the first-best outcome is preserved when a company is privatized. [Zikos, 2007] refutes a challenge to this result based on a case where the privatized firm is still a leader firm after privatization by setting up a subsidy regime of asymmetric subsidies, and thus restores an irrelevance result in this case as well. In chapter 2 of this paper, I consider a particular non-optimal but credible subsidy regime, and different government motivations, and show that in this particular model, a first-best equilibrium is not achievable under privatization.

Classical Economics has also considered the case of Natural Monopolies and Vertical Integration. Consensus seems to be here that if a sector exhibits high fixed costs and low marginal costs, then a state owned monopoly might be the best solution. Breaking up the vertical integration is often also advocated, such as has happened in most Network Industries. [Tomain, 2002] recapitulates the conventional view, and considers how it applies to the Californian electricity transmission network.

1.2 The Theory of the Firm and privatization

Classical economics assumes that a firm is a unitary agent, which can be simply represented by a production function, but this is simply not the case. Anything more advanced than the textbook classic Robinson Crusoe economy will suffer from agency issues, where incentives may not align to a greater or lesser extent. (See for example [Cassidy, 2010] as an extended treatment of behavioural economics, where agency issues are dealt with extensively.) The theory of the firm seeks out to resolve this by considering the relationship between owners, managers and workers. This discipline has reaped many interesting results, such as chapter 5 of [Akerlof and Kranton, 2010], where the two authors consider the impact of productivity when workers do not share the same goals as managers, or [Schmidt, 1996], one of the main references for this paper, where the manager's motivation to work is influenced by the compensation scheme he is offered and "empire building" motivations on his part. In this paper, I introduce a Manager into the Stackelberg privatization game from chapter 2 in chapter 3, and consider what impact this will have.

1.3 Schmidt's Model

As mentioned, [Schmidt, 1996]'s model is one of the main references for this thesis. Below I shortly detail this model.

The model Schmidt proposes is contract theoretical in its approach, meaning that the size of subsidies and wages are handled endogenously between the manager, owner and government. Furthermore, he introduces private information into his model and shows that the government might be interested in *not* knowing the true cost level of the company.

The game proceeds roughly as follows:

- Government chooses whether to nationalize or privatize and subsidize a company.
- A manager is hired.
- He invests a certain effort level into reducing costs.
- The results of this effort are revealed to the owner.
- The owner decides upon a production level, announces a cost level to the government if it does not already know it, and receives the subsidy.

Schmidt's analysis of this game then shows that allocative efficiency will be higher under government ownership, but productive efficiency will be higher under privatization.

The reason for this is found in a game-theoretical credibility argument; the government under nationalization will produce optimally, given costs, and thus the manager will not invest a great deal of effort, while under privatization the government can not know the true cost level of the company, and thus to deter the company from providing a false report of its cost level sets a low subsidy in the high cost case. This spurs the manager to work harder to avoid the lesser rewards arising from lower production in the high cost case.

1.4 Motivations for this paper

Schmidt's model is stark, but fails to take into account that provision of a public good is, even though sometimes legally protected in some cases, not a monopoly in most cases. In Schmidt's model, the company is the only provider of the good, since the government cares only for $b(y)$, the social benefit of the company's production. In real life, close substitutes often exist. For example, the social benefit of an efficient parcel delivery service is similar whether it is the state-owned Post Office providing the service, or a privately held courier company. Many other examples of this exist, including education, transport, security and healthcare, where state-owned companies compete with privately owned companies to a greater or lesser extent.

Thus in my models the Government, still assumed to be perfectly rational and unitary as in Schmidt, takes into account the social benefit of total production of the good. More specifically, in the Stackelberg duopoly model I propose, the government initially owns and operates the dominant company 1, but company 2 also produces the same good. The object of this paper is thus to discuss how the presence of competition in the provision of a public good changes the tradeoffs Schmidt discovers in his paper.

It is important to note that while most of the models in this introduction are very general, showing either that ownership is irrelevant, or that there exists a certain structure where, even under perfectly rational and unitary government, privatization will still bring improvements in productive efficiency, the models I propose may not be as profound. Instead, they seek to illuminate one particular effect in constructed, but hopefully realistic, scenarios.

Chapter 2

The First Model

The first model I propose considers privatization of a dominant firm in a Stackelberg duopoly with similar, but not identical products. Specifically, a situation where Company 1 is initially state owned, and company 2 is privately held. Company 1 is assumed to be a dominant and incumbent company, modelled as a Stackelberg leader, and company 2 is assumed to be a follower company. I use the conventional inverse demand function for similar products, where $b \in (0, 1)$:

$$p(q_i, q_j) = A - q_i - bq_j, \tag{2.1}$$

where $A \in \mathbb{R}^+$ is some predefined constant, $q_i \in \mathbb{R}^+$ is the production of company i and $p(q_i, q_j) \in \mathbb{R}^+$ is the resulting price for company i, for $i = \{1, 2\}$. In both these cases we assume company cost levels c_1, c_2 are public information, but allow them to be dissimilar.

2.1 Reasons for the model

The reasoning behind this model is to be found in the debate surrounding optimal subsidies. [Zikos, 2007] shows that an optimal subsidy does exist for a Stackelberg game as I propose, but under a different cost structure, and where the companies produce the same good. This model does not delve into optimal subsidies, but rather examines a linear subsidy targeted at increasing production of q_1.

The key variable we wish to examine in this model is the effect of competition upon the privatization choice. In this model we assume that the dominant company produces a socially desirable good. The reasons for this particular good being socially desirable are not modelled endogenously, but we might assume that consumption of this good has significant positive externalities for example. We could consider the case of the dominant firm being a railway company, and the follower firm being a coach company. Their productions are close substitutes, but certainly not identical. The government might prefer people to travel by train since these will often be less polluting, but the presence of an extensive coach network is also worth considering, certainly if the marginal cost difference between the two companies is sizeable. Thus, the government is concerned chiefly with promoting adaptation of q_1, and is only interested in q_2 in as far as it is a substitute for q_1.

At this point, I would like to point out a change from convention in the objective function for the government. In this model, based on the argumentation above, the government does not seek to maximise total welfare. Rather, the government seeks to maximise the part of consumer surplus arising from the consumption of $q_1 + bq_2$, that is

$$CS = \frac{(q_1 + bq_2)^2}{2}. \tag{2.2}$$

Furthermore, the government seeks to maximise profits from its involvement in the market, either in the form of state-owned company profits, or total subsidies paid. This seems reasonable if we consider that a government has many functions to handle besides involvement in this sector. Indeed, the government must consider whether the involvement in this sector is to be an income or an expense in the total government balance sheet, and in this particular model, it turns out that the government chooses to impose a tax instead of a subsidy.

Based on the argumentation in the previous paragraph, Consumer Surplus seems a reasonable proxy for the positive benefits arising from consumption of the good. This choice also contributes to the subsidy being chosen negatively, thus leading to a tax.

Returning to the mechanisms of the game, the government thus chooses whether to nationalize or privatize and subsidize. One problem with this approach is that of credibility. The government must be able to commit to providing the subsidy at the stated level. This fact argues against complex subsidy regimes, and brings rise to contract theory, where binding contracts between players are modelled endogenously, as in [Schmidt, 1996]. But in this model, I choose a slightly different approach. I assume that the government can commit to and must choose a linear subsidy of the form $TS = sq_1 + bsq_2$. That is, in the privatization equilibrium $s^*q_1^*$ will be paid to company 1, and $bs^*q_2^*$ to company 2. As well-versed readers might guess, changing the objective function and action space of the government in this way will cause the solution to this model to deviate significantly from the models mentioned in chapter 1, but this will be discussed as the results emerge.

2.2 Description of the model

The model is set up as a dynamic game of complete information, and progresses as follows:

- The government chooses whether to privatize company 1 or keep it nationalized. If the government privatizes the firm, it is sold at auction. The auction process is not specifically modelled, but a reasonable assumption is that it exactly equals expected profits for the private owner, since this is a game of complete information.

- If company 1 is privatized, the government announces a subsidy s per unit of q_1. This subsidy is paid to company 1. Company two also receives a marginal subsidy bs, that is the same as company 1, rescaled by the similarity factor b, for each unit q_2. s is chosen to maximise consumer surplus arising from consumption of $q_1 + bq_2$ minus the costs of the subsidy. We do not explicitly disallow negative subsidies, leaving analysis of these for later.

- Company 1 chooses q_1 so as to maximise company profits and consumer surplus arising from consumption of $q_1 + bq_2$ if nationalized, and profits plus received subsidies if privatized.

- Company 2 chooses q_2 so as to maximise company profits if company 1 is nationalised, and company profits plus received subsidies if company 1 is privatized.

- Payouts are received by G, C_1 and C_2.

2.3 Solution

The solution to this model proceeds by backwards induction to obtain a subgame-perfect Nash eqilibrium. The methodology for this is covered in [Gibbons, 1992].

2.3.1 Nationalization

Examining first the nationalization subgame, we solve for company 2's reaction function.

$$
\begin{aligned}
R_2(q_1) &= \max_{q_2} \left[p(q_1, q_2)q_2 - c_2q_2\right] \\
&= \max_{q_2} \left[(A - q_2 - bq_1)q_2 - c_2q_2\right] \\
&= \frac{A - bq_1 - c_2}{2}, \qquad (2.3)
\end{aligned}
$$

for $q_1 < \frac{A-c_2}{b}$. Company 1 is able to predict this, and thus seeks to solve

$$
\begin{aligned}
q_1^* &= \max_{q_1} \left[p\left(q_1, R_2(q_1)\right) q_1 - c_1q_1 + CS\right] \qquad (2.4) \\
&= \max_{q_1} \left[(A - q_1 - bR_2(q_1))q_1 - c_1q_1 + \frac{(q_1 + bR_2(q_1))^2}{2}\right] \\
&= \max_{q_1} \left[\left(A - q_1 - b\frac{A - bq_1 - c_2}{2}\right) q_1 - c_1q_1 + \frac{(q_1 + b\frac{A-bq_1-c_2}{2})^2}{2}\right] \\
&= \frac{4(A - c_1) - b^3(A - c_2)}{4 - b^4}. \qquad (2.5)
\end{aligned}
$$

Where positive solutions occur when $\frac{A-c_1}{A-c_2} > \frac{b^3}{4}$. Note that in contrast to the classical Stackelberg solution, this implies that if company 1 has very high costs as compared to company 2, and b is also high, company 1 will choose to suspend production while company 2 continues producing. This is though quite unlikely for reasonable choices of A, c_1, c_2.

Substituting (**??**) into company 2's reaction function we get

$$
\begin{aligned}
q_2^* &= R_2(q_1^*) \qquad (2.6) \\
&= \frac{A - b\frac{4(A-c_1)-b^3(A-c_2)}{4-b^4} - c_2}{2} \qquad (2.7) \\
&= \frac{2(A - c_2) - 2b(A - c_2)}{4 - b^4}, \qquad (2.8)
\end{aligned}
$$

where to ensure positivity, we demand $\frac{A-c_2}{A-c_1} > b$. Note that this means if company 2 has significantly higher costs than company 1, company 2 will stop producing for certain high levels of b.

Indeed, consider the case if $q_1 \geq \frac{A-c_2}{b}$, where $R_2(q_1) \leq 0$. Then company

2 will not produce anything, and then company 1 will seek to solve

$$q_1^* = \max_{q_1} \left[p\left(q_1, 0\right) q_1 - c_1 q_1 + CS \right] \tag{2.9}$$

$$= \max_{q_1} \left[(A - q_1) q_1 - c_1 q_1 + \frac{q_1^2}{2} \right]$$

$$= A - c_1. \tag{2.10}$$

Note that we then have $\frac{A-c_2}{A-c_1} \leq b$ by the assumption that $q_1 \geq \frac{A-c_2}{b}$, which implies that $R_2(A - c_1)$ will be zero, and thus this satisfies Nash equilibrium requirements, since each is a best response to the other.

Thus total equilibrium production of $q_1 + bq_2$ if $q_1^* \in [0, \frac{A-c_2}{b})$ is given by

$$q_1^* + bq_2^* = \frac{2(A - c_1) + b(A - c_2)}{2 + b^2}. \tag{2.11}$$

Else, total market production will be

$$q_1^* + bq_2^* = A - c_1. \tag{2.12}$$

2.3.2 Privatization

For the case of privatization, company 2 now receives a subsidy of bs per unit of q_2 produced, so the reaction function thus becomes

$$R_2(q_1) = \max_{q_2} \left[(A - q_2 - bq_1) q_2 - c_2 q_2 + bsq_2 \right]$$

$$= \frac{A + bs - bq_1 - c_2}{2}, \tag{2.13}$$

when $q_1 < \frac{A+bs-c_2}{b}$. Otherwise, as discussed in the previous section, the function is zero.

Company 1 can predict this, and thus solves

$$q_1^* = \max_{q_1} \left[p\left(q_1, R_2(q_1)\right) q_1 - c_1 q_1 + sq_1 \right] \tag{2.14}$$

$$= \max_{q_1} \left[(A - q_1 - b\frac{A + bs - bq_1 - c_2}{2}) q_1 - c_1 q_1 + sq_1 \right] \tag{2.15}$$

$$= \frac{2(A + s - c_1) - b(A + bs - c_2)}{2(2 - b^2)}, \tag{2.16}$$

which is positive for $\frac{A+s-c_1}{A+bs-c_2} > \frac{b}{2}$. Substituting this back into company 2's reaction function gives

$$
\begin{aligned}
q_2^* &= R_2(q_1^*) && (2.17)\\
&= \frac{A + bs - bq_1^* - c_2}{2} && (2.18)\\
&= \frac{(4-b^2)(A+bs-c_2) - 2b(A+s-c_1)}{4(2-b^2)}. && (2.19)
\end{aligned}
$$

As in the nationalization case, we check what happens for $q_1 \geq \frac{A+bs-c_2}{b}$. Then company 1 solves

$$
\begin{aligned}
q_1^* &= \max_{q_1} \left[p\left(q_1, 0\right) q_1 - c_1 q_1 + s q_1\right] && (2.20)\\
&= \max_{q_1} \left[(A - q_1) q_1 - c_1 q_1 + s q_1\right] && (2.21)\\
&= \frac{A + s - c_1}{2}, && (2.22)
\end{aligned}
$$

where for Nash equilibrium, we must check $\frac{A+s-c_1}{2} \geq \frac{A+bs-c_2}{b}$. This reduces to $\frac{b}{2} \geq \frac{A+bs-c_2}{A+s-c_1}$, and it is not as immediately clear as in the nationalization section whether this condition will be satisfied.

Thus total market production if $q_1 < \frac{A+bs-c_2}{b}$ is then

$$
q_1^* + bq_2^* = \frac{2(A+s-c_1) + b(A+bs-c_2)}{4}. \qquad (2.23)
$$

Knowing all this, the Government must set the marginal subsidy s such that

$$
\begin{aligned}
s^* &= \max_s \left[\frac{1}{2}(q_1^* + bq_2^*)^2 - s(q_1^* + bq_2^*)\right] && (2.24)\\
&= \max_s \frac{1}{2}\left(\frac{2(A+s-c_1) + b(A+bs-c_2)}{4}\right)^2 - s\left(\frac{2(A+s-c_1) + b(A+bs-c_2)}{4}\right)\\
&= \frac{(4-2b^2)(A-c_1) + (2b-b^3)(A-c_2)}{(b^2-6)(b^2+2)} && (2.25)
\end{aligned}
$$

But note that for $b \in (0, 1)$, the denominator is negative, and certainly for $A > c_1, c_2$, the numerator will be positive. Thus there is no optimal positive linear subsidy for the government in this game.

The reason for this curious result is to be found in the fact that the two companies exert a considerable degree of market power. Indeed, for

$\sqrt{3} > b > \sqrt{2}$, representing less influence of own production upon price, s^* would be positive.[1]

Note one particular point of concern here, which might make the subsidy less negative, or indeed positive, namely that the government does not take account of the auction price as it plans its subsidy level. One might argue that the auction price is a sunk cost when the subsidy is being planned, but this does seem quite unrealistic. It would be preferable and indeed more realistic if the auction price was also included in (**??**), but this aspect was not considered in time.

At this point the analysis of the privatization subgame changes according to whether we allow a negative subsidy, effectively a tax. If contracts were allowed (and we have already allowed a certain type of contract in which the government commits to offering a linear subsidy), then the tax could certainly be imposed on company 1 as a condition of auction. Indeed, the auction price, which we assumed was equal to expected profits, will depend on whether the subsidy or tax must be accepted, or can be refused.

For the sake of exposition, we consider two situations. One in which the companies will refuse the subsidy if it is negative, and one in which the companies cannot refuse the subsidy: we assume that both companies must accept it.

In the case where the subsidy may be refused, we see that the companies will refuse the optimal subsidy chosen by the government, and their productions will simply correspond to the standard Stackelberg solution, with total market production

$$q_1^* + bq_2^* = \frac{2(A - c_1) + b(A - c_2)}{4}. \tag{2.26}$$

Comparing this with the nationalization case, we see that market production will be less, by a factor $\frac{2+b^2}{4}$.

If though, the companies were forced to accept the subsidy regime offered by the government, the total market production would be

$$q_1^* + bq_2^* = \frac{A(2 + b) - 2c_1 - bc_2 + (2 + b^2)s}{4} \tag{2.27}$$

$$= \frac{2(A - c_1) + b(A - c_2)}{6 - b^2} \tag{2.28}$$

[1]The three roots of the numerator are $b = \{-\sqrt{2}, \sqrt{2}, \frac{-2(A-c_1)}{A-c_2}\}$, where the last one is negative by assumption. An asymptote exists at $b = \sqrt{6}$, and above this, s^* is negative again.

Which is smaller still than the nationalization case and the case where the subsidy is not accepted. But the decrease in consumer welfare is compensated for with the increase in government revenues.

2.3.3 Auction prices under privatization

Since both companies have market power, it would be expected that they generate supernormal profits. Thus, in such a game of complete information as the one we propose, we would expect the auction price to increase until it exactly equals the profits of company 1, ignoring transaction prices.

The profit of company 1, post privatization certainly depends on the presence of the tax. Let us first consider if the tax is not imposed. Then note that the auction price will be

$$AP_P = (A - q_1^* - bq_2^* - c_1)q_1^* \tag{2.29}$$

$$= \frac{(8-2b^2)(A-c_1)^2 - (6b-b^3)(A-c_1)(A-c_2) + b^2(A-c_2)^2}{2(6-b^2)(2-b^2)}. \tag{2.30}$$

If meanwhile the tax is imposed on both companies, then the auction price will instead be

$$AP_T = (A - q_1^* - bq_2^* - c_1)q_1^*$$

$$= \frac{(32+24b^2-16b^4+2b^6)(A-c_1)^2 - (40b-2b^5)(A-c_1)(A-c_2) + 8b^2(A-c_2)^2)}{(6-b^2)^2(2-b^2)(b^2+2)}. \tag{2.31}$$

These numbers are quite difficult to analyse generally, but we see that for both high and low values of b, AP_P will be greater than AP_T, which is exactly as expected.

2.4 Summary and Conclusion

In summary, this model has looked at privatization of a dominant company in a Stackelberg game with non-homogenous products, and concluded that if the government is only able to offer a marginal subsidy per unit, it would rather impose a negative subsidy, i.e. a tax. The government's "revenues" (consumer surplus, company profits and/or tax revenue) in each of the three cases are thus.

For nationalization:

$$
\begin{aligned}
W_N &= (A - q_1^* - bq_2^* - c_1)q_1^* + \frac{(q_1^* + bq_2^*)^2}{2} \\
&= \left(A - \frac{A(2+b)}{2+b^2} + \frac{bc_2 + 2c_1}{2+b^2} - c_1\right) \frac{A\left(4-b^3\right) + b^3c_2 - 4c_1}{4-b^4} + \frac{\left(\frac{A(2+b)}{2+b^2} - \frac{bc_2+2c_1}{2+b^2}\right)^2}{2} \\
&= \left(\frac{b^2(A-c_1) - b(A-c_2)}{2+b^2}\right) \frac{4(A-c_1) - b^3(A-c_2)}{4-b^4} + \frac{(2(A-c_1) + b(A-c_2))^2}{2(b^2+2)^2} \\
&= \frac{4(A-c_1)^2 - 2b^2(A-c_1)(A-c_2) + b^2(A-c_2)^2}{8-2b^4}
\end{aligned}
$$

For privatization where the subsidy is accepted:

$$
\begin{aligned}
W_T &= \frac{(q_1^* + bq_2^*)^2}{2} - s^*(q_1^* + bq_2^*) + AP_T \\
&= \frac{4(A-c_1)^2 + 4b(A-c_1)(A-c_2) + b^2(A-c_2)^2}{2(6-b^2)(b^2+2)} + AP_T. \qquad (2.32) \\
&= \frac{(56 + 16b^2 - 18b^4 + 2b^6)(A-c_1)^2}{(6-b^2)^2(b^2+2)(2-b^2)} \\
&\quad + \frac{-(16b + 8b^3)(A-c_1)(A-c_2) + (14b^2 - 2b^4 - \frac{b^6}{2})(A-c^2)^2}{(6-b^2)^2(b^2+2)(2-b^2)} \qquad (2.33)
\end{aligned}
$$

And if the subsidy is not accepted:

$$
\begin{aligned}
W_P &= \frac{(q_1^* + bq_2^*)^2}{2} + AP_P \\
&= \frac{4(A-c_1)^2 + 4b(A-c_1)(A-c_2) + b^2(A-c_2)^2}{32} + AP_P. \qquad (2.34) \\
&= \frac{(176 - 64b^2 + 4b^4)(A-c_1)^2 - (48b - 16b^3 + 2b^5)(A-c_1)(A-c_2)}{32(6-b^2)(2-b^2)} \\
&\quad + \frac{(28b^2 - 8b^4 + b^6)}{32(6-b^2)(2-b^2)}. \qquad (2.35)
\end{aligned}
$$

Note that the first term of (**??**) will be greater than (**??**) for all b. This is due to $s = 0$ not being the optimal choice of subsidy, and can be verified by noting that the denominator of (**??**) is smaller than the denominator of (**??**) for all $b \in (0, 1)$.

Unfortunately, none of these expressions reduce significantly, but from the final results, we do see that b significantly influences the results for the government.

There is of course a significant inconsistency here in including the auction prices in government utility, when auction prices were not considered when choosing s^*. This means that these results should be treated with care, and (**??**) and (**??**) might be more correct results to look at. Indeed there is a much more obvious linkage between these numbers and the nationalization result W_N.

Turning to the conclusion of this model now. The mechanism of interest was what effect competition would have on the government's decision to privatize. Firstly, there is no value of b that would make privatization superior, which can be verified by checking the inequalities for $b\ in(0,1)$. But even so, b affects the government's welfare significantly.

2.5 Discussion

Our key interest was seeing whether the presence of competition would affect the choice of whether to privatize or not. The short conclusion, based only on the contents of this model, is no. The government never has an incentive to privatize in this model. This is quite obviously due to the form of the government's objective function, and the restriction to linear subsidies.

But a more interesting conclusion of the model is that, in response to a high degree of market power in the production of a socially desirable good, the government might consider imposing a tax instead of a subsidy. (Especially if the auction has already been conducted before the subsidy or tax is being considered.) This is of course partially an effect of the type of subsidy allowed.

Alternative subsidies that might work better could be a "take it or leave it" subsidy that offers a subsidy to the company if and only if it produces the socially desirable quantity. The problem with these subsidy regimes in real life, as is also pointed out in Schmidt, is that they are quite difficult to justify if we consider that the subsidy regime must be specified in advance of production. Or, in our model, it is questionable whether A for example will remain constant between setting the subsidy and production being completed, or whether the (exogenous) externalities that might be of interest change.

An interesting secondary conclusion of this model is that under nationalization, total market production is greater than in the standard Stackelberg game. The mechanism for this is that the leader firm will increase production, but the follower firm's decrease in production is less than the increase from the leader firm.

Chapter 3

The Second Model

In this second model, we expand upon the foundations laid in the first model. As previously, two companies compete in a Stackelberg duopoly with similar products. As before, the government considers whether to nationalize or privatize the dominant company. As before the government chooses a linear subsidy. The difference now is that we introduce a manager to company 1. The manager observes the initial choice by the government, and the subsidy level announced. He then chooses an effort level. For simplicity, we consider only the case of low, medium and high effort levels, and assume that these directly lead with 100% probability to $c_1 \in \{c_H, c_M, c_L\}$, with $c_H > c_2, c_M = c_2$ and $c_L < c_2$ respectively. A key fact is that effort levels and cost levels can only be verified within the firm, following the argument of [Schmidt, 1996].

3.1 Reasons for the model

The previous model in chapter 2 concluded, as expected, that it was difficult to obtain a first-best solution if the government is only restricted to a linear subsidy. In this model, we consider how this changes if we introduce a manager to Company 1. It may be, as in [Schmidt, 1996]'s model, that the manager will cause an increase in productive efficiency when faced with the prospect of his company producing less due to privatization not achieving a first-best solution. And especially interesting will be to see how the manager's effort choice interacts with the similarity factor b.

3.2 Description of the model

The game proceeds as follows:

- The government chooses whether to privatize and subsidize or nationalize the firm.
- If privatized, the government announces a linear subsidy, and then sells the company at auction. Note that here we correct the irregularity from the previous model where the auction price is not included in the government's objective function.
- The manager privately chooses an effort level to maximise his expected utility. The manager's utility is determined by his wage, assumed fixed across all scenarios and thus normalized to 0, and an "empire building" desire, linear in q_1.
- The cost level of company 1 is revealed to the owner (either government or private owner), and production levels are chosen as in the previous model.
- Company 2 does not observe company 1's cost level, but observes q_1 and chooses its own production level as in the previous model.
- Payouts are received by G, C_1, C_2, M.

3.3 Solution

The model thus sketched is an example of a dynamic game of incomplete information, and the conventional solution for these models is a perfect Bayesian Nash equilibrium. The details of this are provided in [Gibbons, 1992]. But the uncertainty about the cost level of company 1 is not as crucial to the game as might be thought, so backwards induction techniques can be partially used.

3.3.1 Nationalization

As in the previous game, company 2 reacts to company 1's choice of quantity:

$$\begin{aligned} R_2(q_1) &= \max_{q_2} \left[p(q_1, q_2) q_2 - c_2 q_2 \right] \\ &= \max_{q_2} \left[(A - q_2 - bq_1) q_2 - c_2 q_2 \right] \\ &= \frac{A - bq_1 - c_2}{2}, \end{aligned} \tag{3.1}$$

Note that company 2 does not care about the cost level of company 1.

Company 1 thus chooses his production level given his cost level

$$\begin{aligned}
q_1(c_1) &= \max_{q_1} \left[p\left(q_1, R_2(q_1)\right) q_1 - c_1 q_1 + CS \right] && (3.2)\\
&= \max_{q_1} \left[(A - q_1 - bR_2(q_1))q_1 - c_1 q_1 + \frac{(q_1 + bR_2(q_1))^2}{2} \right]\\
&= \max_{q_1} \left[\left(A - q_1 - b\frac{A - bq_1 - c_2}{2} \right) q_1 - c_1 q_1 + \frac{(q_1 + b\frac{A-bq_1-c_2}{2})^2}{2} \right]\\
&= \frac{4(A - c_1) - b^3(A - c_2)}{4 - b^4}. && (3.3)
\end{aligned}$$

The manager, knowing this deterministic play of the game after his move, must choose between e_L, e_M and e_H. His utility function is thus

$$U(e_i) = kq_1^*(c_i) - e_i, \tag{3.4}$$

for $i \in L, M, H$, $k \in \mathbb{R}^+$. Since there are only three cases, we can write out the payoff from each, and note that

$$\begin{aligned}
\Delta U(e_L, e_M) &= \frac{4k}{4 - b^4}(c_L - c_M) + (e_L - e_M)\\
\Delta U(e_M, e_H) &= \frac{4k}{4 - b^4}(c_M - c_H) + (e_M - e_H),
\end{aligned}$$

the first term is positive, and the second term is negative. This leads to the simple interpretation that as long as the additional effort is less than the resulting reduction in costs, the manager will be willing to increase his effort level.

3.3.2 Privatization

Under privatization, company 2's reaction function is given by

$$\begin{aligned}
R_2(q_1) &= \max_{q_2} \left[(A - q_2 - bq_1)q_2 - c_2 q_2 + bsq_2 \right]\\
&= \frac{A + bs - bq_1 - c_2}{2}. && (3.5)
\end{aligned}$$

Note that this will depend on c_1 through s, which will depend on the government's belief about c_1.

Company 1 then chooses production level given cost level:

$$q_1(c_1) = \max_{q_1} \left[p\left(q_1, R_2(q_1)\right) q_1 - c_1 q_1 + s q_1 \right] \tag{3.6}$$

$$= \max_{q_1} \left[(A - q_1 - b\frac{A + bs - bq_1 - c_2}{2})q_1 - c_1 q_1 + s q_1 \right] \tag{3.7}$$

$$= \frac{2(A + s - c_1) - b(A + bs - c_2)}{2(2 - b^2)}. \tag{3.8}$$

The manager now chooses his effort level as before to maximise

$$U(e_i) = kq_1^*(c_i) - e_i. \tag{3.9}$$

Our game simplifies quite a bit since s does not depend on the manager's choice of effort level, only on the government's belief about the effort level. And additionally, s is chosen and announced before the effort choice is made. Thus the utility differences are

$$\Delta U(e_L, e_M) = \frac{2k}{4 - 2b^2}(c_L - c_M) + (e_L - e_M)$$

$$\Delta U(e_M, e_H) = \frac{2k}{4 - 2b^2}(c_M - c_H) + (e_M - e_H),$$

which, since $\frac{4}{4-b^2} > \frac{2}{4-2b^2}$ for all $b \in (0, 1)$, means that the manager's effect on production is more effective in the case of nationalization, and he will thus be willing to expend a greater effort under nationalization. This is a curious conclusion, that arises from the independence of s from e_i. In Schmidt's model, the subsidy is dependent on a report of c_1 given by the company, and to prevent lying, the subsidy is intentionally punishing for high cost levels.

This is as far as I am going to take this model, since if I was to try and coerce a similar effect as Schmidt, then a more sophisticated subsidy scheme needs to be modelled. That is what I propose in the next subsection.

3.3.3 Alternative subsidy regimes

Let us examine another subsidy structure. The government in this game, if it decides to privatize the company, has an incentive to cost-effectively encourage the manager to work. We see this in empirical data, where privatization is often held up as a device to increase the productivity of firms. So let us choose a simple subsidy regime, where company 1 provides a report of its cost level to the government, and the government then grants a subsidy based on this. In tune with the spirit of contract bidding, company 1 will receive a linear subsidy of size sq_1 if it reports c_L, $(1 - b)sq_1$ if it reports c_M, and no subsidies if it reports c_H. (We may imagine that company 2 will receive the subsidy if company 1 proves ineffective and if company 2's products are sufficiently similar.)

But even in this case, company 1 will always choose to report a low cost level, no matter what his true type is. This is an example of a so-called Cheap Talk game, where choosing to receive a high marginal subsidy carries no costs for company 1. Thus the incentives for the manager will be the same here as under linear subsidies.

We might now consider a licensing scheme. Under this scheme, company 1 must produce a certain quantity, q_L, or else its operating license will be withdrawn. This scheme would certainly encourage the manager to work, since only if $q_1(c_i) > q_L$ would the company choose to produce. This means that the manager fears liquidation if at least one of the choices of e_i is insufficient to make the company profitable. The problems with this simple licensing scheme are quite clear though. Except perhaps for very high b, it is not subgame rational for the government to shut down the company, and thus it is not a particularly credible subsidy regime.

Chapter 4

The Third Model

The third model presented here is in a slightly different vein than the first two models, in that the similarity factor b is not present. Instead, both companies produce identical goods, and both companies have a manager employed.

4.1 Description of the model

The model I propose is an extension and variation of [Schmidt, 1996], and consists of two companies, numbered 1 and 2. Company 1 is initially state owned, and company 2 is privately held. Company 1 is assumed to be a dominant and incumbent company, and company 2 is assumed to be a follower company. To this end, I model a Stackelberg duopoly where company 1 has the first move, with the conventional inverse demand function

$$p(q_1, q_2) = A - q_1 - q_2, \tag{4.1}$$

where $A \in \mathbb{R}^+$ is some predefined constant, p is the price function, and $q_i \in \mathbb{R}^+$ is the production of company i, for $i = \{1, 2\}$.

Each company furthermore has a type, denoted by c_i^L, c_i^H, representing low, respectively high marginal costs. Fixed costs are assumed zero. The determination of these types will be explained shortly.

The government has a motivation in the game to maximise total welfare. In my model, ignoring externalities, this is modelled as maximising consumer surplus, which with the given price function is simply

$$CS(q_1, q_2) = \frac{(q_1 + q_2)^2}{2}. \tag{4.2}$$

Furthermore, the government (G) takes on the role of C_1 in the nationalization subgame, while in the privatization subgame it constructs a subsidy regime to offer to C_1.

The game proceeds as follows:

- The government chooses whether to nationalize or privatize firm 1.
- If the government privatizes the firm it announces a subsidy regime and then auctions the firm.
- Managers 1 and 2 independently choose their effort levels e_1, e_2. Effort levels are private information.
- Nature chooses the type of company i as outlined above. The type of company i is then revealed to C_i.
- C_1 chooses the production level of company 1, q_1. In the nationalization subgame, he chooses q_1 so as to maximise the sum of company profits and societal benefits of total production, in the privatization subgame, he chooses to maximise company profits and received subsidy.
- Observing q_1 and knowing his own type, C_2 chooses q_2 so as to maximise company profits.

The model thus sketched is an example of a dynamic game of incomplete information, and the conventional solution for these models is a perfect Bayesian Nash equilibrium.

To specify a dynamic game of incomplete information, we need to specify the payoff functions of each agent given a particular type-state of the game. This is easiest done concurrently with the solution.

Under privatization, both companies seek to maximise profits, and do not care about social welfare. But the government may instead choose to provide a subsidy to one or both of the companies to encourage it to maximise social welfare. In Schmidt's model, the subsidy is only given to the one company, but in our model, it is conceivable that the subsidy might be given to both companies, or to the most cost efficient one.

A subsidy scheme must still be credible, and if we allow the government to commit to a subsidy scheme, we can employ the revelation principle, as in Schmidt, to find a corresponding incentive-compatible direct mechanism.

The following solution is incomplete due to time constraints. It is left as is, and following it is a section regarding the intended direction of the solution had time permitted.

4.2 Solution

To solve this model, we tun to the requirements of perfect Bayesian Nash equilibrium (PBNE hereafter). As laid out in Gibbons, a belief and strategy satisfying the following requirements is a PBNE. The requirements are:

R1 At each information set, the player with the move must have a belief about which node has been reached. For non-singleton information sets, this is a probability distribution. For singleton information sets, the belief puts probability 1 on the node.

R2 Given their beliefs, the players' strategies must be sequentially rational.

R3 At information sets on the equilibrium path, beliefs are determined by Bayes' rule and the players' equilibrium strategies

R4 At information sets off the equilibrium path, beliefs are determined by Bayes' rule and the player's equilibrium strategies where possible.

Our first observation is that the nationalization and the privatization subgames can be considered separately.

We thus begin with the nationalization subgame.

4.2.1 Nationalization

We start from the final stage of the game, and examine C_2's move. C_2 seeks to maximise company profits given the information he knows. Since he has already observed his own type and the chosen q_1, he needs to form a belief about what node he is in (whether company 1 has a high or low cost level) and formulate a strategy, that is choice of q_2 for the two of his own types. In the following we use $\mathrm{Pr}_2(\cdot)$ to denote beliefs held by C_2 and $\mathrm{Pr}_1(\cdot)$ to denote beliefs held by C_1.

Thus, when C_2's type is c_k, $k \in \{L, H\}$, he seeks to maximise

$$\max_{q_2} (A - q_1 - q_2 - c_k)\, q_2 \mathrm{Pr}_2(t_1 = c_L) + (A - q_1 - q_2 - c_k)\, q_2 (1 - \mathrm{Pr}_2(t_1 = c_L)), \tag{4.3}$$

but note that, in a Nash eqilibrium, C_1 will be acting rationally, and C_2 knows this, so from the choice of q_1 and knowing all other relevant quantities, $p_{2,t_1=c_L}$ will either be believed to be 1 or 0 by Bayes' rule, and the maximisation problem reduces to the standard Stackelberg reaction problem, where the solution is thus characterised by the reaction function

$$R_2(q_1) = \frac{A - q_1 - c_k}{2}. \tag{4.4}$$

For company 1 of type c_j, in the nationalization subgame, C_1 thus seeks to maximise

$$\max_{q_1} \left(\left(A - q_1 - q_2 - c_j \right) q_1 + CS \right) \Pr_1(t_2 = c_L) + \left(\left(A - q_1 - q_2 - c_j \right) q_1 + CS \right) \left(1 - \Pr_1(t_2 = c_L) \right), \tag{4.5}$$

where the Bayesian Nash equilibrium choice of q_1 will be the best response to the optimal choice of q_2, which is given by $R_2(q_1)$. Thus we rewrite the equation as

$$\begin{aligned} q_1^* = \max_{q_1} & \left[\left(A - q_1 - \frac{A - q_1 - c_L}{2} - c_j \right) q_1 + b \left(q_1 + \frac{A - q_1 - c_L}{2} \right) \right] \Pr_1(t_2 = c_L) \\ & + \left[\left(A - q_1 - \frac{A - q_1 - c_H}{2} - c_j \right) q_1 + b(q_1 + \frac{A - q_1 - c_H}{2}) \right] (1 - \Pr_1(t_2 = c_L)), \end{aligned} \tag{4.6}$$

The solution to this is given by

$$q_1^* = \frac{A + b}{2} - c_j + \frac{c_L \Pr_1(t_2 = c_L) + c_H(1 - \Pr_1(t_2 = c_L))}{2}, \tag{4.7}$$

which reduces to the standard Stackelberg solution $\frac{A-c}{2}$ if $c_k = c_H = c_L, b = 0$.

Inserting this into $R_2(q_1)$ we get the optimal strategy of O_2 as,

$$q_2^* = R_2(q_1^*) = \frac{A - b}{4} - \frac{c_j}{2} + \frac{c_k}{2} - \frac{c_L \Pr_1(t_2 = c_L) + c_H(1 - \Pr_1(t_2 = c_L))}{4}, \tag{4.8}$$

where we have previously argued that the presence of c_j need not concern us, since this private information is implicitly revealed in q_1. Note that this solution again reduces to the standard Stackelberg solution when $c_k = c_j = c_H = c_L, b = 0$. Note one curious thing though, that all else equal, as b increases, then the total quantity produced in the market increases as well. This is similar to the effect observed in chapter 2.

Before turning to analyse the managers, let us characterise these strategies a bit further. If C_1 is of type c_L, his production, given a belief $\Pr_1(t_2 = c_L)$ will be

$$q_{1,c_L,p}^* = \frac{A + b}{2} - c_L \frac{2 - \Pr_1(t_2 = c_L)}{2} + c_H \frac{1 - \Pr_1(t_2 = c_L)}{2}, \tag{4.9}$$

meanwhile, if he is of type c_H, his production will be

$$q_{1,c_H,p}^* = \frac{A + b}{2} - c_H \frac{1 + \Pr_1(t_2 = c_L)}{2} + c_L \frac{\Pr_1(t_2 = c_L)}{2}. \tag{4.10}$$

As long as $c_H > c_L$, then $q^*_{1,c_L,p} > q^*_{1,c_H,p}$ for any valid $\Pr_1(\cdot)$, as expected. Furthermore, as $\Pr_1(t_2 = c_L)$ increases, then the chosen $q^*_{1,c_i,p}$ will decrease for both types.

For C_2, we can now also justify what we have said earlier: C_1 signals his type to C_2 unambiguously, since there is no $\Pr_1(t_2 = c_L) \in [0,1]$ for which $q^*_{1,c_L,p} = q^*_{1,c_H,p}$, this would require $c_H = c_L$.

Thus it is most interesting to outline C_2's strategy for his own types along with the cases where he believes $\Pr_2(t_1 = c_L) = 0 \Pr_2(t_1 = c_L) = 1$, since these are the only two beliefs as required by Bayesian Nash equilibrium that are consistent with the requirements.

$$q^*_{2,c_L,1} = R_2(q^*_{1,c_L,p}) = \frac{A-b}{4} - \frac{c_L \Pr_1(t_2 = c_L) + c_H(1 - \Pr_1(t_2 = c_L))}{4}, \tag{4.11}$$

and

$$q^*_{2,c_L,0} = R_2(q^*_{1,c_H,p}) = \frac{A-b}{4} - \frac{c_H}{2} + \frac{c_L}{2} - \frac{c_L \Pr_1(t_2 = c_L) + c_H(1 - \Pr_1(t_2 = c_L))}{4}, \tag{4.12}$$

and

$$q^*_{2,c_H,1} = R_2(q^*_{1,c_L,p}) = \frac{A-b}{4} - \frac{c_L}{2} + \frac{c_H}{2} - \frac{c_L p_{t_2=c_L} + c_H(1 - p_{t_2=c_L})}{4}, \tag{4.13}$$

and

$$q^*_{2,c_H,0} = R_2(q^*_{1,c_H,p}) = \frac{A-b}{4} - \frac{c_L \Pr_1(t_2 = c_L) + c_H(1 - \Pr_1(t_2 = c_L))}{4}. \tag{4.14}$$

Here we note the interesting simliarity between these strategies, and mention again the caveat that $\Pr_1(t_2 = c_L)$ is contained within q_1.

We are yet to describe $\Pr_1(t_2 = c_L)$ and $\Pr_2(t_1 = c_L)$ properly though, and for this we need to turn to the equilibrium strategies of the two managers.

Staying in the Nationalization subgame, M_2 and M_1 seek to maximise their own expected utility. In this case, they must have a belief about the choice of the other manager, which in turn will define the weighting of the four situations in the subsequent owner's turns.

Thus M_1, M_2 seeks to maximise their own utility, where they must take account of the other manager's choice, and the effort cost and influence of effort on cost level. Unfortunately time does not allow me to continue this analysis in detail.

4.2.2 Privatization

In privatization extra stages of the game are added. The government announces a subsidy scheme, and then auctions off the firm. The interesting thing here as compared to the previous models is that the auction price cannot be determined exactly and fairly, since the expected profit is not known. The managers and companies then take their turns, and the subsidy is paid out.

4.2.3 Outline of the remaining solution

As can be glimpsed from the nationalization case, there is a rich interplay between the two managers. Furthermore in the privatization subgame, the optimal government subsidy and auction price will also have to be determined. Both of these things will be affected by the expected effort level of the managers.

Chapter 5

Conclusion and scope for further research

This report has set out to consider the effects of competition and managerial incentives surrounding privatization. Three models have been presented, and two of them solved. The models do not deliver clear-cut conclusions, due partly to the difficulty in choosing the form of subsidies that the government offers, and partly also in choosing suitable government objectives.

5.1 Summary of results

For the first model, discussed in chapter 2, the effect of competition was examined. Here it was shown that under the specific choice of utility functions for the agents, and restricting the government to choosing only linear subsidy regimes, that a first-best solution was not achievable for privatization. A curious result was that the subsidy might be chosen negatively, to be a tax instead, but this results rests upon the problematic lack of the revenue from the auction in the government's objective function.

The main result of the model is that of the effect of competition. Increasing b under nationalization will lead to an increase in government welfare, but also, if company 1 is very inefficient as compared to company 2, then increasing b might make company 1 stop producing totally. The same is much more unlikely in the privatization cases.

The second model considered was an extension of the first, adding a manager to company 1. The expected effect was that the manager would work harder to ensure higher productions under privatization, but under the linear subsidy scheme from the first model, the manager would work harder under nationalization than under privatization. A few other subsidy schemes were also considered, but none of these would cause a truth-telling equilibrium where the company reported its true cost level, thus prompting the manager to work harder. Applying the revelation principle to determining the subsidy would likely lead to the desired effect, but at the cost of sacrificing the applicability of the model to studying certain simple, empirically observed subsidy schemes.

The third model is ambitions in its scope, and suppresses certain things from the first two models while emphasising other factors. It needs further work on choosing the form of the manager's utility functions especially, and after that a quite complicated construction of a perfect Bayesian Nash equilibrium.

5.2 Further research

Suggestions for expansions and modifications of the three models have already been presented in the main model chapters, but even so a few of them are worth reiterating.

Firstly, reworking the subsidy calculations in chapter 2 would perhaps actually cause the government to choose a subsidy instead of a tax. This type of model would be more in tune with what we expect. Also in this model, it would be interesting to look more specifically at whether the subsidy could be used to internalise externalities, as discussed in the reasons for the model.

Modelling a credible, simple subsidy scheme in tune with the Stackelberg model is important to drawing more interesting conclusions from chapter 3. This subsidy scheme should indirectly encourage the manager to work harder, either through the private owner of the company or even by treating directly with the manager. It is also important to develop a realistic utility function for the manager, and modelling the wage agreement between manager and owner endogenously might also bring more interesting results. Contract theory would be very applicable for this.

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